Spanwise Growth of Vortex Structure in Wall Turbulence

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Recent studies of the structure of wall turbulence have lead to the development of a conceptual model that validates and integrates many elements of previous models into a relatively simple picture based on self-assembling packets of hairpin vortex eddies. By continually spawning new hairpins the packets grow longer in the streamwise direction, and by mutual induction between adjacent hairpins the hairpins are strained so that they grow taller and wider as they age. The result is a characteristic growth angle in the streamwise-wall normal plane. The spanwise growth of individual packets implies that packets must either merge or pass through each other when they come into contact. Direct numerical simulations of the growth and interaction of spanwise adjacent hairpins shows that they merge by the vortex connection mechanism originally proposed by Wark and Nagib (1990). In this mechanism the quasi-streamwise legs of two hairpins annihilate each other, by virtue of having opposite vorticity, leaving a new hairpin of approximately double the width of the individuals. PIV measurements in planes parallel to the wall support this picture. DNS of multiple hairpins shows how the spanwise scale doubles when the hairpins form an array.

Key Words : Turbulence, Vortices, Wall Turbulence, Structure, Eddies

1. Introduction

One of the most famous empirical 'laws' in turbulence is v. Karman's law which states that in a certain region above a wall the mean velocity varies as a logarithmic function of the distance from the wall. The inverse constant of proportionality is the v. Karman constant. The constant applies for a variety of flows - boundary layer, pipe, channel, smooth wall, rough wall- and its value is independent of the Reynolds number. The logarithmic variation is used to set a wall boundary condition in the vast majority of all turbulent CFD solutions of flows involving solid boundaries. Acceptance of the logarithmic law is so wide spread that a recent challenge to its validity (Barrenblatt and Chorin, 1997) has stimulated furious experimental activity to resolve the issue, despite relatively small practical ramifications. To claim to understand wall turbulence, it is essential, at the minimum, to give a complete explanation for the logarithmic variation and a sound prediction of the value of v. Karman's constant. But, there exists at this time no theory capable of predicting the constant. This is, perhaps, not surprising since the v. Karman constant is most likely an embodiment of fundamental aspects of the structure of the eddies in wall turbulence, and a unified mechanistic picture of the near-wall turbulence structure and generation process has only recently begun to emerge (Adrian, Meinhart and Tomkins, 2000).

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2. Hairpin Vortex Packet Model of Wall Turbulence

Experimental and computational results for smooth walls (Meinhart and Adrian 1995, Zhou,

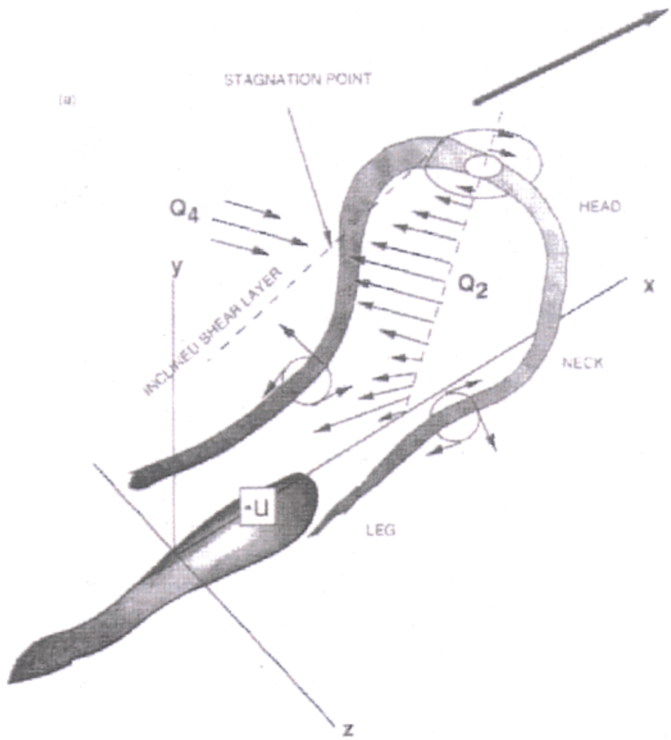

Fig. 1 Paradigm of a single hairpin vortex

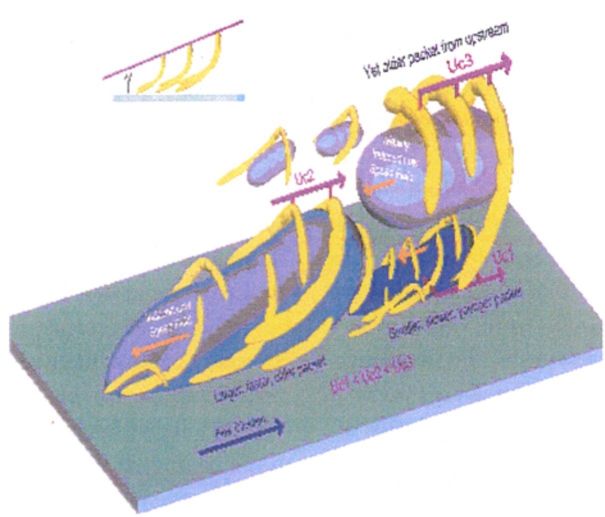

Fig. 2 Paradigm of a hierarchy of hairpin packets

Adrian and Balachandar, 1996, 1999, Tomkins, Adrian and Balachandar 1998, Adrian, Meinhart and Tomkins, 2000) give strong support to a (undoubtedly simplistic) mechanistic picture of wall turbulence based on a hierarchy of hairpin packets. The central element in this model is the hairpin vortex, Fig. 1. This elementary form of eddy has been suspected of playing a major role in wall turbulence since the pioneering work of Theodorsen (1952). A second central element in

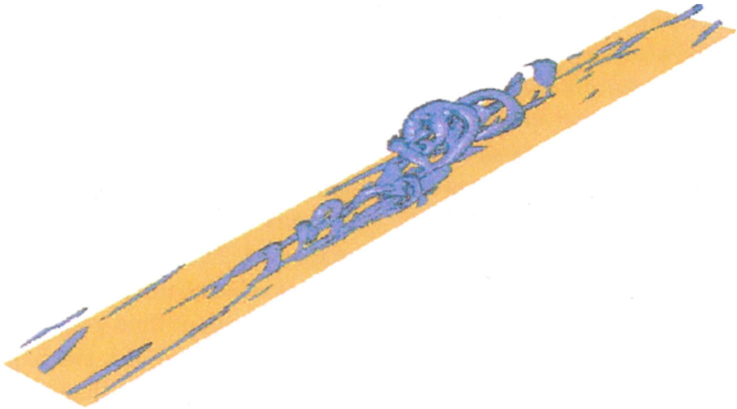

Fig. 3 Complex hairpin packet that evolves from a slightly asymmetric initial disturbance. Such packets are also thought to evolve from local bumps on a wall

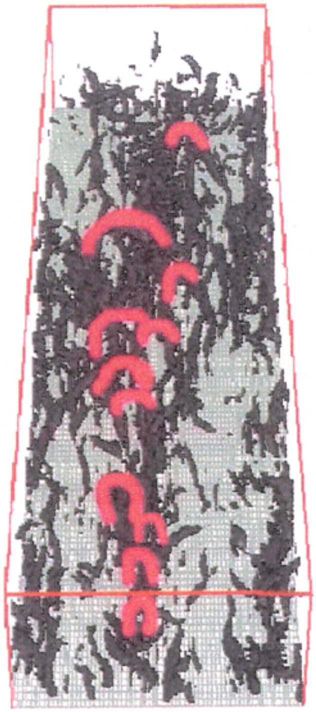

Fig. 4 Turbulent eddies in Reynolds number=300 channel flow. Vortices that are part of a hairpin packet have been highlighted to make them visible amongst the background clutter. Note the similarity between the groups of eddies in this fully turbulent flow and the complex hairpin packet in Fig. 3

the anatomy of wall turbulence is the packet of hairpins, consisting of a group of hairpins more or less aligned in the streamwise direction. Observations of hairpins in groups were first made by flow visualizations using smoke (Head and Bandyopadhyay, 1981) and H_2-bubbles (Smith, 1984). Persuasive demonstration of the pervasiveness and importance of hairpins and hairpin packets had to await the development of DNS and multi-point experimental methods which enabled the quantitative studies of Zhou, *et al*. (1999) and Adrian, *et al*. (2000).

The third central element in this picture is an autogeneration process whereby a hairpin vortex spawns a younger, smaller hairpin, which in turn spawns a still younger hairpin, and so on (Smith, *et al.*, 1991, Zhou, *et al.*, 1999). Repeated autogeneration forms packets of hairpins lined up behind one another, Fig. 2. The grand-sire hairpin is the largest, with the younger generations following behind, each generation being smaller. The growth rate of the hairpins, the time between regeneration of new hairpins and their convection velocity combine to give the envelop of each packet a characteristic growth angle that is believed to be one factor that determines v. Karman's constant.

Observation of these structures, in fully three-dimensional time-dependent DNS or in two-dimensional snapshots from laser sheet experiments, is intrinsically difficult, partly due to chaos, partly due to subtle technical difficulties in visualizing eddies (or even defining them), and partly due to the complexity that results from many generations of hairpin packets coexisting and interacting. For example, Fig. 3 shows a packet that evolves out of a single hairpin with a small ($\sim 10^{-2}$) initial asymmetry. While the packet paradigm is simple, the realizations are not. In fact, the eddies of a fully turbulent channel flow field, Fig. 4, contain hairpin packets that look not significantly more complex than the example in Fig. 3.

The method of visualizing the eddies in Fig. 3 and 4 is called the 'swirling strength'. Vorticity is a good indicator of eddies when it is concentrated in a small-diameter core. This is because the velocity field associated with a concentrated vortex is amenable to interpretation in terms of the Biot-Savart law. If the vorticity is associated with distributed shear, similar interpretation is not so useful. To detect concentrated vorticity having the nature of a core, several procedures have been developed based on critical point theory (Perry and Chong 1987, Chong, Perry and Cantwell, 1990, Hunt, Wray and Moin, 1988, Jeong and Hussain 1995). The method we prefer uses the imaginary part of the complex eigenvalue of the velocity gradient tensor (Zhou, *et al.* 1999) called the 'swirling strength'. In regions where all the eigenvalues of the velocity gradient tensor are real, the swirling strength is zero. The swirling strength has a very simple kinematic interpretation that makes it useful as a new kinematic quantity. Specifically, a nonzero swirling strength indicates local dominance of rotation-rate over strain-rate. Then, in a frame of reference moving at local velocity, there exists a plane on which the projected path of fluid particles spirals in or out. The reciprocal of the swirling strength is the period required for a fluid particle to orbit the point at which the velocity gradient tensor is evaluated. In pure shear flow, the period of the orbit is infinite, and the swirling strength is zero, despite the non-zero vorticity of the flow. Hence, visual renderings of surfaces of non-zero swirling strength show only those vortical regions of the flow that 'swirl', while discriminating against the regions of strong shear.

3. Growth of the Spanwise Length Scale

The logarithmic law is believed to be a consequence of an orderly property that underlies the chaos of wall turbulence wherein the eddies have a self-similar geometrical structure characterized by a length scale that is proportional, in a statistical sense, to distance from the wall. Large Re implies a thick logarithmic layer and if the model is to be taken seriously, a thick logarithmic layer must imply existence of large hairpin packets whose size scales with the layer thickness. Recent observations of large hairpin packets in the atmospheric boundary layer provide important support for the model.

The lateral spacing of $\lambda_z^+ \cong 100$ between near-wall low-speed streaks is well established. This spacing can also be interpreted as the lateral size of the youngest generation of hairpins that form in the near-wall region. The single-most important problem is to explain the sequence of events that allows the very small hairpin eddies created in the low Reynolds number region near a wall to grow into eddies that are orders of magnitude larger while maintaining structural similarity.

Strict similarity requires growth of length scales in all three directions Imagine a hairpin vortex that grows in all directions while maintaining exactly the same shape. A brief glance at Fig. 3 and 4 is sufficient to convince one that such perfect self-similar growth will never occur. Instead, the similarity must of a statistical nature. In the terminology of Townsend (1976) we must seek 'self-preserving growth'. Thus, as the packets age, we look for integral length scales that grow in x, y and z in constant proportion.

The growth of an individual hairpin along the streamwise and wall-normal directions can be explained relatively easily. The mean shear stretches the hairpin along the streamwise direction, while the effect of self-induction is to curl the hairpin backwards and lift the head away from the wall (see Zhou, *et al.* 1999 for details). A balance between shear- and self-induced stretching is observed to result in a self-similar growth of the hairpin over time.

Numerical simulations of the evolution of a single hairpin vortex have shown that as a hairpin at the wall grows older it generates subsequent secondary and tertiary hairpins, and so on, which are aligned along the streamwise direction (Zhou *et al.* 1999). The heads of the sequence of older to younger hairpins that form a packet, are observed to form a characteristic angle with respect to the wall that ranges between 12 and 20 degrees with a mean of 13 to 15 degrees. PIV measurements on the streamwise-wall-normal plane by Adrian, *et al.* (2000) provide experimental support for similar growth of the hairpins at much higher Reynolds numbers. The more complex pattern of asymmetric vortices shown in Fig. 3 also grows upwards at about the same rate, indicating that the growth is not sensitive to details of the vortex structure. The packet in Fig. 3 also grows in the spanwise direction at a rate roughly similar to that found in the wall-normal direction, as do the packets that can be found in Fig. 4. Further studies are needed to establish the spanwise growth rate firmly, but at this point it can be safely stated that the available evidence (Kempka 1988, Moin, *et al.*, 1986) is not inconsistent with the hypothesis of three-dimensional self-preserving growth of the eddies in a packet.

While self-preserving growth of a single packet might explain the logarithmic law as a consequence of the scale increasing in proportion to y, it cannot be the full story. Lateral interaction between hairpins must be an important ingredient in the spanwise scaling of the hairpin vortices as they grow along the streamwise and wall-normal directions. As the packets expand in the spanwise direction they must ultimately interact by vortex encounters. Encounters also occur due to larger, faster packets running over smaller, slower packets. But, DNS results (some of which will be presented below) indicate that these are not so dramatic or influential as lateral encounters. Some such possible vortex packet encounters are depicted schematically in Fig. 5. In lateral encounters, the opposing vorticity in adjacent legs of two hypothetically identical hairpins could annihilate them, resulting in a larger hairpin of the same height, but double the width of the original hairpins. Figure 5(a) shows schematically such a lateral vortex merger resulting in larger hairpins having twice the spanwise spacing of $\lambda_z^+ \cong 100$. Further lateral merging of the larger hairpins can lead to subsequent progressive increase in spanwise scale. (To the author's knowledge this sort of lateral hairpin vortex pairing was first proposed by Wark and Nagib (1990), although Perry and Chong (1982) must be credited with proposing the generic concept of "vortex pairing of two eddies in one hierarchy to form an eddy in the next hierarchy".) We therefore envision the growth of scale in the wall layer to occur both by continuous expansion of the eddies in an individual packet and the merger of eddies in adjacent packets.

Figure 5(a) depicts a scenario where there is perfect symmetry along the spanwise direction, while Fig. 5(b) shows a more realistic scenario where perfect spanwise symmetry is not present. Vortex reconnection and merger still apply, and spanwise growth of the hairpin can be anticipated. In interpreting this frame it must be cautioned that the wall-normal elevation of the hairpin varies over its length and from hairpin to hairpin. Also, it has been established that taller

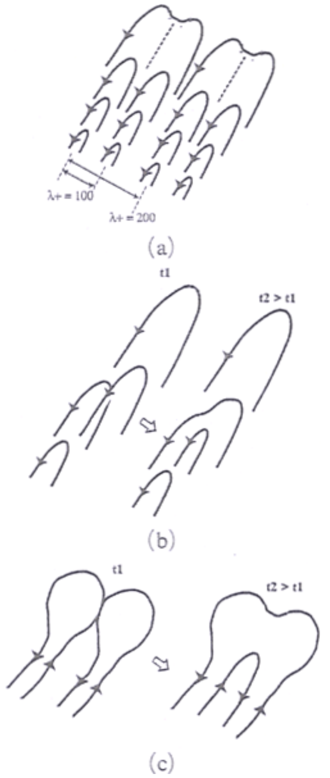

Fig. 5 Spanwise growth by hairpin vortex pairing. (a) Two similar hairpins pairing symmetrically produce a new hairpin whose spanwise width is approximately twice the original width; (b) a faster moving upstream hairpin encounters an offset slower moving hairpin. The vortex cut and reconnection leads to a larger hairpin, similar to case (a) and a smaller hairpin having the same circulation; (c) Two similar, symmetrically positioned omega-shaped vortices connect to form a larger hairpin plus a smaller hairpin having opposite circulation

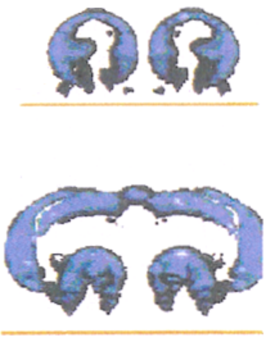

Fig. 6 A doublewide hairpin is formed due to lateral encounter of two hairpins. The adjacent legs of two hairpins are annihilated by the opposite vorticity

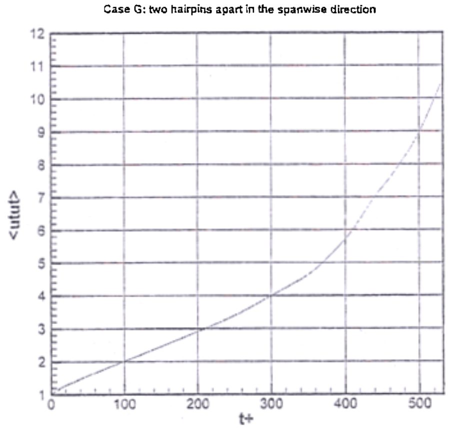

Fig. 7 After the lateral encounter of two hairpins the kinetic energy of the streamwise velocity increases more rapidly as a result of a reduction of back-induction by the hairpins

hairpin packets move faster in the streamwise direction than shorter ones. Thus, Fig. 5(b) considers a taller upstream packet that travels faster and catches up with the shorter downstream packet. Vortex reconnection occurs at the point of intersection of the vortex cores, and the exact location depends upon the geometry and size of each hairpin.

Observations of fully turbulent DNS results suggest that other scenarios of spanwise growth can also be conjectured, such as that shown in Fig. 5(c). In the early stages of their development the hairpins vortices are observed to have a Ω-shaped head (Zhou *et al.* 1999). Consequently, the first intersection of two hairpins may occur at the outermost sections of the Ω. Interestingly, the vortex merger of the form shown in Fig. 5(c) results in an inner hairpin whose direction of rotation is opposite to the normal hairpin. A spanwise vortex merger of this nature can be seen in Fig. 6, wherein an end-view of two merging vortices from a DNS is shown.

Such vortex rearrangements have a strong influence on near-wall statistics. For example, in Fig. 6 as a result of the vortex merger, the back-induction of the hairpin is reduced substantially, leading to a sudden increase in the kinetic energy of the streamwise velocity, which is shown in Fig. 7. The effect is to increase drag. Thus, the manner in which the interactions occur is important.

There is also experimental evidence to support the hypothesis that hairpin vortex packets grow by lateral merger. Tomkins (2000) has performed a series of PIV measurements in planes parallel to a smooth wall in a boundary layer wind tunnel. The purpose was to see if the parallel plane data were consistent with the occurrence of packets of vortices, especially at Reynolds numbers higher than the values that are currently accessible via DNS. Power spectra in the spanwise wavenumber and linear stochastic estimates of the streak patterns each showed conclusively that the spanwise spacing of the streaks grows in proportion distance above the wall. The vector pattern results also showed that merging of two low-speed streaks into a larger low-speed streak. It has been well established that the long low-speed streaks are associated with the streamwise-aligned hairpins in a packet. Therefore the merger of long streamwise streaks could be interpreted as a manifestation of the pairing of two packets of hairpins.

Direct numerical simulations of multiple hairpin packets interacting demonstrate several possible consequences for the evolution of the packets. Two such scenarios are shown in Fig. 8. In Fig. 8(a) and 8(b) the perspective and side views of a complex vortex structure is shown. This structure evolved from an initial condition consisting on five hairpins, four of which were placed at the corners of a rectangle, while the last one placed at the center of the rectangle. The hairpin vortices were allowed to evolve and interact in a background turbulent channel flow of $Re_{\tau}=300$. The vortices evolve and interact in a complex manner and the resulting vortex structure after some evolution is shown in the figure. Evidence of five original hairpin vortices forming a rectangular pattern appears to have been forgotten, indicating considerable spanwise interaction

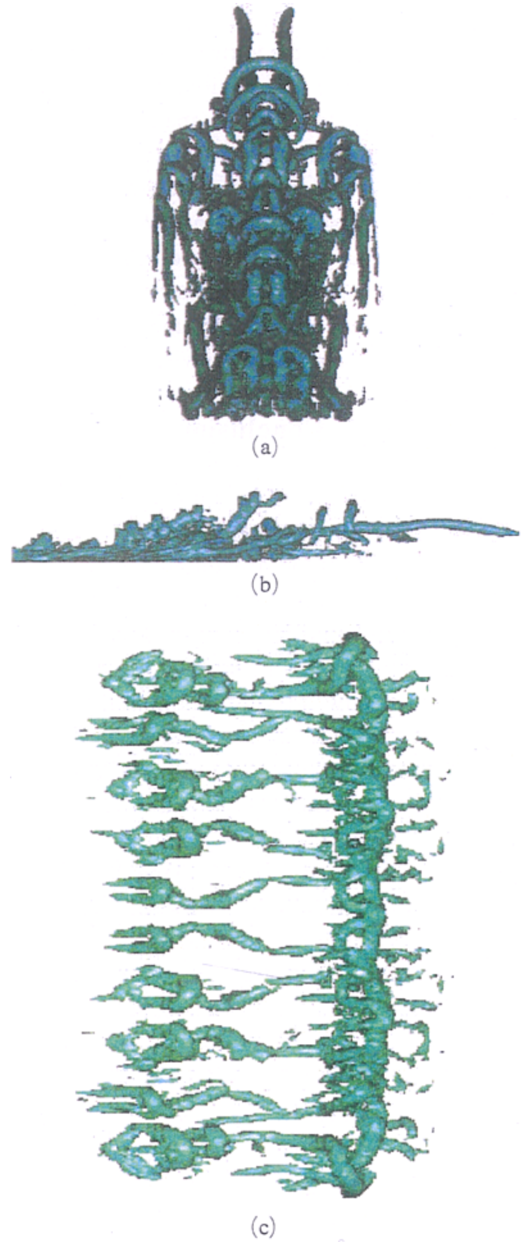

Fig. 8 Interactions of multiple hairpin packets in a mean turbulent flow at $Re_{\tau}=300$. (a) and (b) are the perspective and side views of the vortical structure resulting from interactions of five hairpins, four of them are located at each corner of a rectangle and one in the center of it. (c) is the top view of ten interacting hairpins aligned in the lateral direction. After pairing the ten hairpins form five groups, each of them developed from spanwise interaction

and growth.

Figure 8(c) shows the vortex structure resulting from the interaction of ten initial hairpin vortices placed side-by-side along the spanwise direction in a turbulent channel flow of Re_{τ}.=300. The figure shows the status of the vortex structure after evolution for a short duration. While the ten vortices are clearly visible at the upstream location, the downstream development is influenced by spanwise interaction with the ten hairpins forming five groups after spanwise pairing.

4. Conclusions

The celebrated logarithmic law and v. Karman constant are most likely an embodiment of fundamental aspects of the structure of eddies in wall turbulence. A simple conceptual model for the self-similar growth of the hairpin vortex eddies is presented. By continually spawning new hairpins the packets grow longer continuously in the streamwise direction. As a result of mean shear and mutual induction the hairpins are strained, and they grow longer, taller and wider as they age. The result is a characteristic growth angle in the streamwise-wall normal plane and spread angle in the streamwise-spanwise plane. The spanwise growth of individual packets implies that they must either merge or pass through each other when they come into contact. Several scenarios of spanwise growth of the hairpins through vortex annihilation and reconnection are conjectured. Results from direct numerical simulations of the growth and interaction of spanwise adjacent hairpins show that the hairpins merge by a vortex connection mechanism similar to that originally proposed by Wark and Nagib (1990). In this mechanism the quasi-streamwise legs of two hairpins annihilate each other, by virtue of having opposite vorticity, leaving a new hairpin of approximately double the width of the individuals. Thus, scale growth occurs continuously for a time, until packets and/or hairpins encounter one another, then the scale increases very rapidly to approximately double the width in the spanwise direction. The time for viscous vortex reconnection is so small as to approximate a discontinuity on the time scale of the evolution of the packets.

While the scenario of continuous/discontinuous growth is complicated, it establishes a clear mechanism by which the scales in the x- and z-direction can be proportional to the scales in the y-direction. This is the essential component needed to predict a logarithmic variation. The rate of variation, as embodied in v. Karman's constant remains to be related to the details of the vortex growth and interaction.

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